Practica 5

Derivadas parciales.

Problema 1

En los siguientes casos Halle las derivadas:

$$\frac{dw}{dt}$$

a) $w = x^2 + y^2,\ x = \cos(t),\ y = \operatorname{sen}(t);\ t = \pi$

b) $w = \frac{x}{z} + \frac{y}{z},\ x = \cos^2(t),\ y = \operatorname{sen}^2(t),\ z = \frac{1}{t};\ t = 3$

c) $w = \ln\left(x^2 + y^2 + z^2\right),\ x = \cos(t),\ y = \operatorname{sen}(t),\ z = 4\sqrt{t};\ t = 3$

Ejercicio 1

a) $$\frac{\partial w}{\partial x} = 2x, \ \frac{\partial w}{\partial y} = 2y \quad (1)$$

$$\frac{dx}{dt} = -\text{sen}(t), \ \frac{dy}{dt} = \cos(t) \quad (2)$$

(1,2) ➡ $$\frac{dw}{dt} = -2x\text{sen}(t) + 2y\cos(t) = -2\cos(t)\text{sen}(t) + 2\text{sen}(t)\cos(t) = 0$$

b) $$w = \frac{x}{z} + \frac{y}{z} = \frac{\cos^2(t)}{\frac{1}{t}} + \frac{\text{sen}^2(t)}{\frac{1}{t}} = t \qquad (1')$$

(1') ➡ $$\frac{dw}{dt} = 1 \Rightarrow \frac{dw(3)}{dt} = 1$$

problema 2.

Hallar las derivadas $\frac{\partial z}{\partial u}$ y $\frac{\partial z}{\partial v}$, en los casos que se muestran a continuación:

a) $z = 4e^{x}\ln(y), \;\; x = \ln\left(u\cos(v)\right), \;\; y = u\text{sen}(v); \;\; (u, v) = (2, \frac{\pi}{4})$

b) $z = 4\,e^{x}\,\text{tg}^{-1}(\frac{x}{y}), \;\; x = u\cos(v), \;\; y = u\,\text{sen}(v); \;\; (u, v) = \left(1.3, \frac{\pi}{6}\right)$

Ejercicio 2

a)

i) $$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u} = (4e^x \ln y)\left(\frac{\cos v}{u \cos v}\right) + \left(\frac{4e^x}{y}\right)(\sin v) = \frac{4e^x \ln y}{u} + \frac{4e^x \sin v}{y}$$

$$= \frac{4(u \cos v)\ln(u \sin v)}{u} + \frac{4(u \cos v)(\sin v)}{u \sin v} = (4 \cos v)\ln(u \sin v) + 4 \cos v$$

$\left(2, \frac{\pi}{4}\right)$ evaluando $\longrightarrow$ $= \sqrt{2}(\ln 2 + 2)$

$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v} = (4e^x \ln y)\left(\frac{-u \sin v}{u \cos v}\right) + \left(\frac{4e^x}{y}\right)(u \cos v) =$$

$$= -(4e^x \ln y)(\tan v) + \frac{4e^x u \cos v}{y}$$

$$= [-4(u \cos v)\ln(u \sin v)](\tan v) + \frac{4(u \cos v)(u \cos v)}{u \sin v}$$

$$= (-4u \sin v)\ln(u \sin v) + \frac{4u \cos^2 v}{\sin v}$$

evaluando $\longrightarrow$ $\left(2, \frac{\pi}{4}\right)$ $= -2\sqrt{2}\ln 2 + 4\sqrt{2}$

ii) $z = 4e^x \ln y = 4(u \cos v) \ln(u \sin v)$

$$\Rightarrow \frac{\partial z}{\partial u} = (4 \cos v) \ln(u \sin v) + 4(u \cos v)\left(\frac{\sin v}{u \sin v}\right)$$

$$= (4 \cos v) \ln(u \sin v) + 4 \cos v$$

$$= \sqrt{2}(\ln 2 + 2)$$

evaluando

$$\left(2, \frac{\pi}{4}\right)$$

$$\frac{\partial z}{\partial v} = (-4u \sin v) \ln(u \sin v) + 4(u \cos v)\left(\frac{u \cos v}{u \sin v}\right)$$

$$= (-4u \sin v) \ln(u \sin v) + \frac{4u \cos^2 v}{\sin v}$$

evaluando

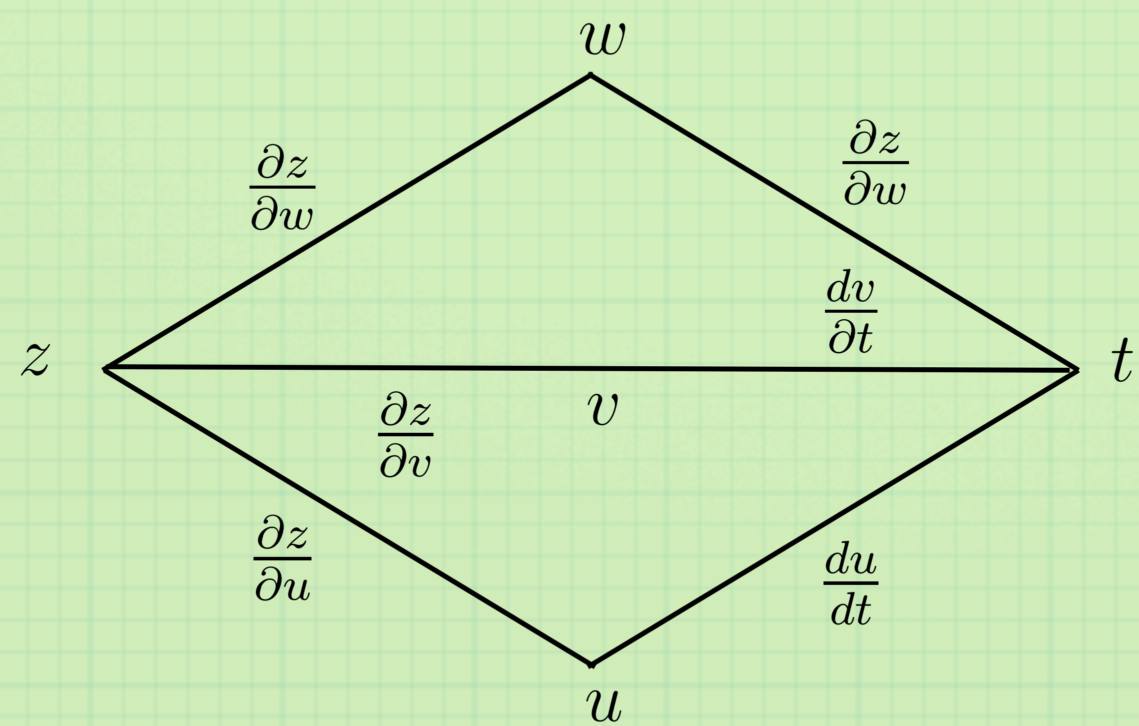
$$= -2\sqrt{2} \ln 2 + 4\sqrt{2}$$

Ejercicio 3.

Escriba el diagrama de arbol, y la formula de la regla de la cadena para la siguiente derivada:

$$\frac{dz}{dt}, \text{ para } z = f(u, v, w),\ u = g(t),\ v = h(t),\ w = k(t)$$

(r)



Problema 3.

a) Suponga que la siguiente ecuación define a la variable z como una función continua de las variables (x,y),

$$xy + z^3x - 2yz = 0$$

Entonces calcule:

$$\frac{\partial z}{\partial x}, \text{ en el punto } (1,1,1)$$

$$y + \left(3z^2 \frac{\partial z}{\partial x}\right) x + z^3 - 2y \frac{\partial z}{\partial x} = 0$$

$$\left(3z^2 x - 2y\right) \frac{\partial z}{\partial x} = -y - z^3$$

$$\frac{\partial z}{\partial x} = \frac{-y - z^3}{(3z^2 x - 2y)}$$

Esta formula es equivalente al calculo que hemos efectuado

$$\frac{\partial z}{\partial x} = \frac{-\frac{\partial F(x,y,z)}{\partial x}}{\frac{\partial F(x,y,z)}{\partial z}}$$

evaluando en $(1,1,1)$ $\quad \frac{\partial z}{\partial x} = -2$

Ejercicio 3.

a) Suponga que la siguiente ecuación define a la variable x como una función continua de las variables (y,z),

$$xz + y\ln(x) - x^2 + 4 = 0$$

Entonces calcule:

$$\frac{\partial x}{\partial z}, \text{ en el punto } (1, -1, -3)$$

Ejercicio 4.

Considere la función diferenciable

$$f(u, v, w)$$

con

$$u = x - y$$
$$w = z - x$$
$$v = y - z$$

Demuestre que f(u,v,w), satisface

$$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0$$